Generalized linear transport theory in dilute neutral gases and dispersion relation of sound waves

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The transport processes in dilute neutral gases are studied by using the kinetic equation with a collision relaxation model that meets all conservation requirements. The kinetic equation is solved keeping the whole anisotropic part of the distribution function with the use of the continued fractions. The conservative laws of the collision operator are taken into account with the projection operator techniques. The generalized heat flux and stress tensor are calculated in the linear approximation, as functions of the lower moments, i.e., the density, the flow velocity and the temperature. The results obtained are valid for arbitrary collision frequency ν with the respect to kv_t and the characteristic frequency ω, where k^{-1} is the characteristic length scale of the system and v_t is the thermal velocity. The transport coefficients constitute accurate closure relations for the generalized hydrodynamic equations. An application to the dispersion and the attenuation of sound waves in the whole collisionality regime is presented. The results obtained are in very good agreement with the experimental data.

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I. INTRODUCTION

The study of the spatial and temporal nonlocal transport effects in neutral gases is motivated by fundamental problems of statistical physics [1] such as the connection between generalized versions of hydrodynamics and generalizations of Grad's theory [2], as well as by practical problems such as the dispersive properties of sound waves, the properties of shock structures in weakly collisional gases and the spectral response of fluids with the use of the scattering of the light and the neutrons by fluid disturbances.

Numerous studies of physical phenomena in neutral gases are based on the hydrodynamical models. Validity conditions of these hydrodynamical equations are often not fulfilled, particularly when the inhomogeneity scale length L and the characteristic frequency ω of the physical phenomena is comparable respectively to the particle mean-free path λ and to the collision frequency ν. The classical diffusionlike equations for the momentum and energy transfer (the so-called hydrodynamic regime) are no longer correct and spatial and temporal nonlocal effects should be also accounted for under these conditions.

In the literature several works have been devoted to the derivation of the generalized Boltzmann and hydrodynamic equations. By generalized equations, we mean that the transport coefficients in the Fourier space depend on the frequency ω and the wave number $k=2\pi/L$. Although this physical problem is an old problem, it remains in our knowledge still open. Several models are proposed to derive kinetic and hydrodynamic generalized equations for dilute and dense gases. Hereafter we present a brief review for these papers.

First we review the papers related to the structure factor which allows us to investigate the effects of the molecular dynamics on the transport properties. Among the pioneering work in this field, Yip and Nelkin [3] have computed the space and time-dependent density correlation function $G(\vec{r},t)$ [the measured quantity in light and neutron scattering experiments is its Fourier transform $S(\vec{k},\omega)$] in dilute fluids by using the Bhatnagar-Gross-Krook (BGK) kinetic model [4]. They have shown in particular that in the Knudsen regime (weakly collisional gas) the hydrodynamic description is no longer valid. In Ref. [5], the correlation function for a classical dense fluid is calculated from the linearized Vlasov equation. The effective interatomic potential is taken into account and the results obtained show similarity with inelastic neutron scattering experiments. In the seventeen's, Furtado, Mazenko, and Yip [6] have improved the kinetic model based on the Boltzmann-Enskog equation to compute in a dense hard-sphere fluid, the dynamic structure factor $S(\vec{k},\omega)$. In particular, they approximate the collision operator in the Enskog equation and assume a k-dependent hard-sphere diameter. The results obtained are in good agreement with the neutron inelastic scattering measurements. A similar work has been reported in Ref. [7] where the collision operator is approximated by the BGK model [4]. In Ref. [8] the neutron scattering function for hard sphere have been computed by using molecular dynamics simulation. It has been shown that the Enskog kinetic and hydrodynamic theories give qualitatively accurate results in describing the thermal fluctuations as long as the scale length L is greater than the particle mean-free path λ. Using standard projection operator techniques Kirkpatrick [9] demonstrated the connection between the work of Refs. [7,8].

On the other hand more recently some papers have been devoted to the calculation of the generalized hydrodynamic equations. Velasco and Garcia Colin [10] have proposed a model of generalized transport coefficients for dilute gases using the Grad's method [2] for solving the Boltzmann equation up to 26 moments. The linearized transport coefficients for weakly nonlocal regime are calculated. In Ref. [11], the model of Ref. [10] is extended to moderately dense gases and the Enskog equation is solved with the use of the 13-moment Grad method. The calculation is developed up to the second order in the density expansion.

Moreover, Alexeev [12] proposed a generalized Boltzmann equation based on the BBGKY-hierarchy taking into account three time-scales, namely the mean time of particle interaction and the usual collision mean-free time and typical hydrodynamic time. The three generalized hydrodynamic equations (conservation of mass, momentum and energy) are established. These equations could be used to study the turbulence theory on the Kolmogorov scale.

In Ref. [13], a generalized macroscopic equation model is derived from the Boltzmann equation with the BGK collision operator. The solution of the kinetic equation is obtained using a modified Chapman-Enskog [14] expansion. The distribution function is expanded up to the first order with respect to the small parameter $\varepsilon \sim \lambda/L$ and the solvability conditions are not applied at each order of the expansion. This procedure yields results valid in weakly nonlocal range.

In this work we calculate from the Boltzmann equation the generalized linear transport coefficients in dilute neutral gases for arbitrary collisionality parameter $\frac{\lambda}{L}$ and for arbitrary normalized phase velocity $\frac{\omega}{kv_t}$, where $v_t=\sqrt{\frac{T}{m}}$ is the thermal velocity, T is the temperature in energy units (used throughout this work), and m is the particle mass. In the literature, the usual approach to solve the Boltzmann equation is the Chapman-Enskog [14] method. This method is based on the expansion of the kinetic equation on the collisionality parameters λ/L and ν/ω. The first term of the expansion is the local Maxwellian. To solve the kinetic equation, the expansion is truncated at a given order. This method, robust to describe systems very close to the thermodynamic equilibrium, breaks down to solve weakly collisional systems. The main argument to explain the failure of these methods is that the collisions are not enabled to ensure the Maxwellianization of the isotropic part of the distribution function. The isotropic part has to be treated on equal footing with the other terms of the expansion.

In the present work we present an alternative approach to solve the Boltzmann equation keeping the whole anisotropic part of the distribution function with the use of the continued fractions [15] and the projection operators [16] to ensure the conservative properties of the collision operator. In Sec. II we present the kinetic model. Section III is devoted to the solution of the Boltzmann equation and to the computation of the generalized transport coefficients. An application devoted to the sound waves in neutral gases is presented in Sec. IV. Finally we give in a last section a discussion and a summary of this work.

II. EQUATIONS OF THE MODEL

The starting point of this work is the Boltzmann equation for a monatomic particle gas

$$\frac{\partial f_g}{\partial t} + \vec{v} \cdot \frac{\partial f_g}{\partial \vec{r}} = C(f_g) \tag{1}$$

where $f_g(\vec{v},\vec{r},t)$ is the distribution function of the particle gas and the right-hand side is the collision operator [or the time rate of change of $f_g(\vec{v},\vec{r},t)$ due to collisions]. The other variables have their usual meaning. An infinite system of moment equations can be obtained by taking different velocity moments of the Boltzmann equation. The turbulence in the system is assumed negligible and the correlations of waves are not taken into account. For weakly turbulent systems for instance, the quasilinear theory could be applicable. The untruncated hierarchy of such equations is completely equivalent to the Boltzmann equation and the all kinetic information is not lost in such a hierarchy. Usually, the infinite system is truncated to the first three equations, for the density $n(\vec{r},t)$, the flow velocity $\vec{V}(\vec{r},t)$, and the temperature $T(\vec{r},t)$. The higher order moments, namely the heat flux $\vec{q}(\vec{r},t)$ and the stress tensor $\Pi_{ij}(\vec{r},t)$ in one-constituent gases, computed from the kinetic theory in terms of the lower moments, constitute the closure relations for these fluid equations. It is obvious that the validity range of the closure relations imposes the validity range of the fluid model.The resulting fluid equations are the density, the momentum, and the energy transport equations

$$\frac{\partial n}{\partial t} + \frac{\partial nV_i}{\partial x_i} = 0, \tag{2}$$

$$\frac{\partial V_i}{\partial t} + V_j\frac{\partial V_i}{\partial x_j} = -\frac{1}{nm}\frac{\partial (nT)}{\partial x_i} - \frac{1}{nm}\frac{\partial \Pi_{ij}}{\partial x_j}, \tag{3}$$

$$\frac{\partial}{\partial t}\left(\frac{nmV^2}{2} + \frac{3nT}{2}\right) + \frac{\partial}{\partial x_j}\left[\left(\frac{nmV^2}{2} + \frac{5nT}{2}\right)V_j + \Pi_{ij}V_i + q_j\right] = 0, \tag{4}$$

where

$$\Pi_{ij}(\vec{r},t) = m\int\left((v_i - V_i)(v_j - V_j) - \frac{(\vec{v}-\vec{V})^2}{3}\delta_{ij}\right)f_g d\vec{v}, \tag{5}$$

and

$$\vec{q}(\vec{r},t) = \int \frac{1}{2}m(\vec{v}-\vec{V})^2(\vec{v}-\vec{V})f_g d\vec{v}. \tag{6}$$

We note that Eqs. (2)–(4) conserve respectively the density, the momentum, and the energy of the particles and they do not involve collisional terms, given that, the following conservative laws have to be fulfilled

$$\int C[f_g(\vec{v})]d\vec{v} = 0, \tag{7}$$

$$\int m\vec{v}C[f_g(\vec{v})]d\vec{v} = 0, \tag{8}$$

$$\int \frac{1}{2}mv^2C[f_g(\vec{v})]d\vec{v} = 0. \tag{9}$$

In the following, it is more convenient to rewrite Eq. (1) using the random velocity, $\vec{v}\,'=\vec{v}-\vec{V}$. For this, one computes the total time derivative of $f_g(\vec{v}\,',\vec{r},t)$ and uses the free force approximation, $\frac{d\vec{v}\,'}{dt}=-\frac{d\vec{V}}{dt}$, obtaining

$$\frac{\partial f_g(\vec{v}',\vec{r},t)}{\partial t} + (\vec{v}' + \vec{V})\cdot\frac{\partial f_g(\vec{v}',\vec{r},t)}{\partial \vec{r}} - \left[\frac{\partial \vec{V}}{\partial t} + (v_i' + V_i)\frac{d\vec{V}}{dx_i}\right]\cdot\frac{\partial f_g(\vec{v}',\vec{r},t)}{\partial \vec{v}'} = C(f_g). \quad (10)$$

In this paper we shall concentrate on situations where the gas is near thermal equilibrium constituted by a reference thermal (global) state defined by a temperature T_0, a density n_0 and no mean velocity $\vec{V}_0=0$, and a perturbed state defined by the hydrodynamic variables, $n(\vec{r},t)$, $\vec{V}(\vec{r},t)$, and $T(\vec{r},t)$. In addition, for the sake of simplicity, we assume that all the inhomogeneities are in the x direction, i.e., $n=n(x,t)$, $T=T(x,t)$, and $\vec{V}=V(x,t)\hat{x}$.

The basic equation of our model is the perturbed Boltzmann equation with respect to the global equilibrium. For this purpose we separate the distribution function into a global Maxwellian and a perturbed distribution function, i.e., $f_g(\vec{v}',x,t)=F_M(v',n_0,T_0)+f(\vec{v}',x,t)$. The collision operator is modeled by a relaxation operator [4] (or the BGK operator) with a constant collision frequency ν, however we consider that this operator relaxes the distribution function towards a local perturbed Maxwellian. Thus its expression reads

$$C(f) = \nu[f_M(v',x,t) - f(\vec{v}',x,t)] \quad (11)$$

where

$$f_M(v',x,t) = \frac{n(x,t)}{n_0}\mu_0\exp\left(-\frac{v'^2}{2v_t^2}\right) + \frac{T(x,t)}{T_0}\mu_0\left(\frac{v'^2}{2v_t^2} - \frac{3}{2}\right)\times\exp\left(-\frac{v'^2}{2v_t^2}\right) \quad (12)$$

is the perturbed Maxwellian, $\mu_0=n_0/(2\pi v_t^2)^{3/2}$ and $v_t=(T_0/m)^{1/2}$ is the background thermal velocity. The hypothesis of a constant collision frequency is required to fulfil the conservative properties (7) and (9). We will see in Sec. IV, devoted to computation of the dispersion relation of sound waves, that the collision frequency ν can be estimated by comparison with experimental data in the collisional range ($\lambda/L \ll 1$). Using these approximations the perturbed Boltzmann equation is readily obtained

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} + V\frac{\partial F_M}{\partial x} - \frac{\partial V}{\partial t}\frac{\partial F_M}{\partial v_x} - v_x\frac{dV}{dx}\frac{\partial F_M}{\partial v_x} = \nu(f_M - f), \quad (13)$$

where for clarity the "prime" in the random velocity is removed. Due to the axial symmetry of the problem along the x axis, in the velocity space the distribution function depends on the coordinates (v,μ), i.e., $f(\vec{v},x,t)=f(v,\mu,x,t)$, where $\mu=\cos\theta=v_x/v$. To solve Eq. (13), first we perform its Fourier transform $(x\leftrightarrow k, t\leftrightarrow\omega)$, after that, we expand the distribution function and Eq. (13) on the Legendre polynomial basis $[P_n(\mu)]$, obtaining

$$\tilde{f}(\vec{v},\omega,k) = \sum_{n=0}^{\infty} P_n(\mu)\tilde{f}_n(y,\omega,k), \quad (14)$$

$$-i\omega\tilde{f}_0 + ikv_t\sqrt{2/3}y^{1/2}\tilde{f}_1 + \nu\tilde{f}_0 = \nu\tilde{f}_M - \frac{2}{3}ikv_t y\mu_0\frac{\tilde{V}}{v_t}\exp(-y), \quad (15)$$

$$-i\omega\tilde{f}_1 + ikv_t y^{1/2}\sqrt{\frac{2}{3}}\tilde{f}_0 + ikv_t\frac{2\sqrt{2}}{\sqrt{15}}y^{1/2}\tilde{f}_2 + \nu\tilde{f}_1 = -i\omega\frac{\tilde{V}}{v_t}\sqrt{\frac{2}{3}}y^{1/2}\mu_0\exp(-y), \quad (16)$$

$$-i\omega\tilde{f}_2 + \frac{2\sqrt{2}}{\sqrt{15}}ikv_t y^{1/2}\tilde{f}_1 + \nu\tilde{f}_2 + \frac{3\sqrt{2}}{\sqrt{35}}ikv_t y^{1/2}\tilde{f}_3 = -\frac{4}{3\sqrt{5}}y\mu_0\exp(-y)ik\tilde{V}, \quad (17)$$

$$-i\omega\tilde{f}_{n+1} + \frac{n+1}{\sqrt{(2n+1)(2n+3)}}ik\sqrt{2}v_t y^{1/2}\tilde{f}_n + \nu\tilde{f}_{n+1} + \frac{n+2}{\sqrt{(2n+3)(2n+5)}}ik\sqrt{2}v_t y^{1/2}\tilde{f}_{n+2} = 0, \quad (n > 1), \quad (18)$$

where $y=mv^2/2T_0$ and the notation "~" means that the corresponding quantities are written in the Fourier space. To derive Eqs. (15)–(18), we have used the recursive formula [17]

$$\mu P_n(\mu) = \frac{n}{\sqrt{(2n-1)(2n+1)}}P_{n-1}(\mu) + \frac{n+1}{\sqrt{(2n+1)(2n+3)}}P_{n+1}(\mu), \quad (19)$$

to compute explicitly the second term in the left-hand side of Eq. (13).

In Ref. [16], the approach presented to solve the kinetic equations is based on the projection operator P and its orthogonal complement, Q. In particular, the authors have calculated explicitly the expressions of these operators for the relaxation collision operator (11). We give in Appendix A, a simplified presentation of these mathematical techniques and a summary of the calculation of the projectors P and Q associated to the relaxation operator. Equation (15) which accounts for the collisional properties (7) and (9) is obtained by multiplying it by the operator Q

$$(-i\omega+\nu)(\tilde{f}_0-\tilde{f}_M)+\frac{\sqrt{2}}{\sqrt{3}}ikv_t y^{1/2}\tilde{f}_1 = \frac{\sqrt{2}}{\sqrt{3}\Gamma(3/2)}ikv_t\left[\left(\frac{5}{2}M_1^1-M_1^2\right)+\left(-M_1^1+\frac{2}{3}M_1^2\right)y\right]\exp(-y), \qquad (20)$$

where $\Gamma(x)$ is the Euler function and where we have used the notation, $M_n^m=\int_0^\infty y^m\tilde{f}_n dy$.

Now we can go a step further and use the mathematical results derived in Ref. [15] to solve the infinite set of Eq. (18). We can see that the component $\tilde{f}_2$ is expressed in terms of $\tilde{f}_3$ and $\tilde{f}_4$, then, $\tilde{f}_3$ is expressed in terms of $\tilde{f}_4$ and $\tilde{f}_5$, and so on. The solution of this algebraic system of equations can be found with the use of the continued fractions [15]. It results the expression

$$\tilde{f}_3=F_1\tilde{f}_2. \qquad (21)$$

Here F_1 (and F_0, F_2 which will be defined in the next section) are infinite continued fractions defined by following recursive formula calculated in Ref. [15]

$$F_n=\left[-i\omega+\nu+\frac{(n+1)^2}{4(n+1)^2-1}2k^2v_t^2yF_{n+1}\right]^{-1}, \qquad (22)$$

where F_n is an infinite continued fraction of order n, and it incorporates the contributions from all the Legendre modes with $n>1$. We should note that expression (21) is the exact solution of the infinite set of Eq. (18). We pointed out that the techniques of the continued fractions were previously used in the transport theory. For instance, Mori [18] and Nagano *et al.* [19] have reported a general development of the kernel of a generalized Langevin equation in terms of a continued fraction expansion and in Refs. [20,21], the diffusion coefficient, the heat conductivity, and the shear viscosity are expanded on the continued fractions with respect to the frequency and the wave vector.

Equations (16), (17), (20), and (21) constitute the basic equations of this work. They correspond to a set of linear algebraic equations for the functions $\tilde{f}_0-\tilde{f}_3$ with source terms expressed with respect to the generalized forces: the gradients of density and temperature, and the flow velocity. They are exactly equivalent to the perturbed Boltzmann equation (13) coupled to the conservative properties (7)–(9). Therefore these equations are valid in the most general ordering, corresponding to arbitrary values of the relevant parameters $\frac{\nu}{kv_t}$ and $\frac{\omega}{kv_t}$.

III. GENERALIZED TRANSPORT COEFFICIENTS

To proceed further, we have to solve Eqs. (16), (17), (20), and (21) for the perturbed distribution functions $\tilde{f}_0-\tilde{f}_3$. From Eqs. (16), (17), and (21) we deduce the first and the second anisotropic functions that we need to compute the transport coefficients

$$\tilde{f}_1=-\sqrt{\frac{2}{3}}v_t y^{1/2}F_1 ik\tilde{f}_0-\frac{8\sqrt{2}}{15\sqrt{3}}v_t y^{3/2}F_1F_2\mu_0\exp(-y)k^2\tilde{V}+\frac{\sqrt{2}}{\sqrt{3}v_t}y^{1/2}F_1\mu_0\exp(-y)i\omega\tilde{V}, \qquad (23)$$

$$\tilde{f}_2=-\frac{4}{3\sqrt{5}}v_t^2yF_1F_2k^2\tilde{f}_0-\frac{4}{3\sqrt{5}}\nu yF_1F_2\mu_0\exp(-y)ik\tilde{V}. \qquad (24)$$

Substituting expression (23) into the isotropic equation (20), we readily obtain the expression of the symmetric distribution function

$$\tilde{f}_0=(\nu-i\omega)F_0\tilde{f}_M-\frac{2}{3}i\omega\mu_0yF_0F_1\exp(-y)ik\tilde{V}+\frac{16}{45}k^2v_t^2\mu_0y^2F_0F_1F_2\exp(-y)ik\tilde{V}+\frac{2\sqrt{2}}{\sqrt{3\pi}}ikv_tF_0\exp(-y)\left(\frac{5}{2}M_1^1-M_1^2\right)+\frac{4\sqrt{2}}{3\sqrt{3\pi}}ikv_tF_0y\exp(-y)\left(-\frac{3}{2}M_1^1+M_1^2\right). \qquad (25)$$

Then from Eqs. (23)–(25), the two first anisotropies of the distribution function can be written as

$$\tilde{f}_1=-\sqrt{\frac{2}{3}}ikv_t(\nu-i\omega)y^{1/2}F_0F_1\tilde{f}_M+\sqrt{\frac{2}{3}}i\omega\mu_0y^{1/2}F_1\times\exp(-y)\left(1-\frac{2}{3}k^2v_t^2yF_0F_1\right)\frac{\tilde{V}}{v_t}+\frac{2\sqrt{2}}{\sqrt{3\pi}}y^{1/2}F_0F_1\times\exp(-y)\left(\frac{5}{2}M_1^1-M_1^2\right)+\frac{4\sqrt{2}}{3\sqrt{3\pi}}y^{3/2}F_0F_1\times\exp(-y)\left(-\frac{3}{2}M_1^1+M_1^2\right)+\frac{8}{15}\sqrt{\frac{2}{3}}ikv_t\mu_0y^{3/2}F_1F_2\times\exp(-y)\left(1-\frac{2}{3}k^2v_t^2yF_0F_1\right)ik\tilde{V}, \qquad (26)$$

and

$$\tilde{f}_2=-\frac{4}{3\sqrt{5}}k^2v_t^2(\nu-i\omega)yF_0F_1F_2\tilde{f}_M+\frac{8}{3\sqrt{45}}i\omega k^2v_t^2\mu_0y^2\times F_0F_1^2F_2\exp(-y)ik\tilde{V}-\mu_0yF_1F_2\exp(-y)\times\left(\frac{4}{3\sqrt{5}}\nu+\frac{64}{135\sqrt{5}}k^4v_t^4y^2F_0F_1F_2\right)ik\tilde{V}-\frac{8\sqrt{2}}{3\sqrt{15\pi}}k^2v_t^2ikv_tyF_0F_1F_2\exp(-y)\left(\frac{5}{2}M_1^1-M_1^2\right)-\frac{16\sqrt{2}}{9\sqrt{15\pi}}k^2v_t^2ikv_ty^2F_0F_1F_2\exp(-y)\left(-\frac{3}{2}M_1^1+M_1^2\right). \qquad (27)$$

We can now compute from Eqs. (26) and (27) the desired generalized transport expressions

$$\tilde{q}_x = \frac{8\pi}{\sqrt{3}} m v_t^6 M_1^2 \tag{28}$$

for the x component of the heat flux and

$$\tilde{\Pi}_{xx} = \frac{16\pi\sqrt{2}}{3\sqrt{5}} m v_t^5 M_2^{3/2} \tag{29}$$

for the x-x component of the stress tensor. From Eqs. (5) and (6), we can see that the remaining components of $\tilde{q}_i$ and $\tilde{\Pi}_{ij}$ vanish. The explicit expressions of Eqs. (28) and (29) are relegated for the sake of clarity in Appendix B. We have found that the heat flux involves linear combination of Fourier components of the temperature gradient $(\nabla\tilde{T})_k = ik\tilde{T}(\omega,k)$, the perturbed flow velocity, $\tilde{V}(\omega,k)$, whereas the stress tensor involves the combination of the perturbed temperature $\tilde{T}(\omega,k)$ and the spatial derivative of the flow velocity $\left(\frac{\partial\tilde{V}}{\partial x}\right)_k = ik\tilde{V}(\omega,k)$, i.e.

$$\tilde{q}_x(\omega,k) = -K_T n_0 T_0 v_t \frac{ik}{|k|}\frac{\tilde{T}}{T_0} - \alpha_V n_0 T_0 \tilde{V}, \tag{30}$$

$$\tilde{\Pi}_{xx}(\omega,k) = -\alpha_T n_0 \tilde{T} - \mu n_0 T_0 \frac{ik}{|k|}\frac{\tilde{V}}{v_t}. \tag{31}$$

In Eqs. (30) and (31) the dimensionless transport coefficients are given in Appendix B through Eqs. (B4)–(B7), where K_T is the thermal conductivity, α_V is the convective heat flux coefficient, α_T is the temperature anisotropy coefficient, and μ is the viscosity coefficient. Notice that the coefficients α_V and α_T have no counterparts in the local relations, i.e., they vanish in the limit $\nu \to \infty$.

Now we present the numerical computations of the transport coefficients. As an illustration we give in Figs. 1–3 the real and imaginary parts of these coefficients as functions of the normalized phase velocity $\xi = \frac{\omega}{\sqrt{2}|k|v_t}$. We can see that the transport coefficients decrease with increasing parameter $\frac{\nu}{\omega}$ and they tend to zero for $\xi \to \infty$. In the limit of strong collisionality, the Taylor expansion of the continued fractions with respect to the small parameters, $\frac{kv_t}{\nu} \sim \frac{\omega}{\nu} \sim \varepsilon \ll 1$, leads to $F_n \approx \frac{1}{\nu}\left(1 + i\frac{\nu}{\omega}\right)$. Hence, the integrals $Y_{n,m}^{i,j} = \int_0^\infty y^n \tilde{F}_0^m \tilde{F}_1^i \tilde{F}_2^j dy$, defined in Appendix B become Euler functions and we can easily recover at the lowest order on $\frac{\omega}{\nu}$, the collisional coefficients, $K_T = \frac{5}{2}\frac{|k|v_t}{\nu}$, $\mu = \frac{4}{3}\frac{|k|v_t}{\nu}$, and $\alpha_V = \alpha_T \approx 0$. On the other hand, for finite values of ω/ν and for $\frac{\nu}{kv_t} \ll 1$, the transport coefficients due to the nonlocal effects, present nonvanishing asymptotic behavior. We have also checked that the generalized Onsager symmetry, $\alpha_T = \alpha_V$ is fulfilled. The nonlocal quantities (30) and (31) can be expressed in terms of convolution products in the spatiotemporal space, involving the temperature gradient and the fluid velocity, and kernels defined by the set of nonlocal transport coefficients (B4)–(B7).

Rewritten in the (t,x) space, the transport coefficients can be used as closure relations for the hydrodynamic equations (2)–(4). Rigorously, these relations are valid to close linear hydrodynamic equations. However it is well known that the linear transport coefficients can be still used to close nonlinear hydrodynamic equations and a discussion on this approximation is given in Sec. V. We should emphasize that the closure relations (30) and (31) are valid for arbitrary collisionality and arbitrary characteristic time and space scales.

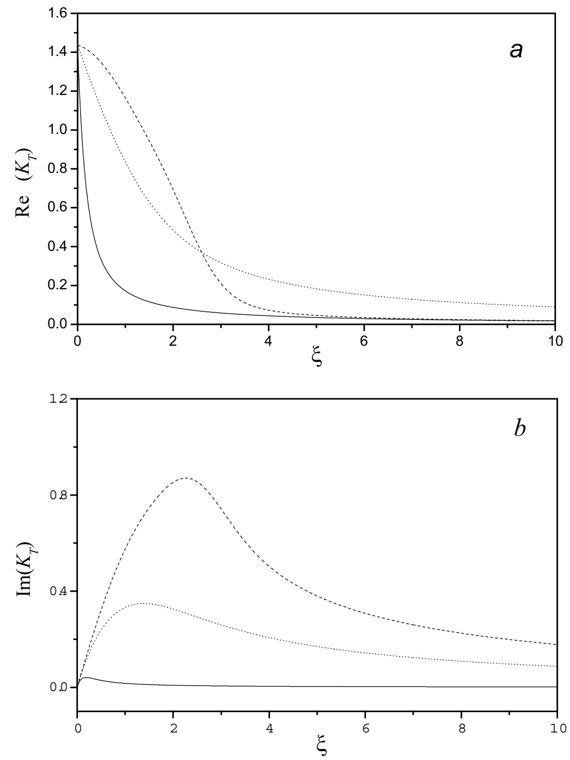

FIG. 1. Normalized thermal conductivity K_T as a function of the normalized phase velocity $\xi = \frac{\omega}{\sqrt{2}kv_t}$ for different collisionality parameter $\frac{\nu}{\omega} = 0.1$ (dashed curve), $\frac{\nu}{\omega} = 1$ (dotted line), and $\frac{\nu}{\omega} = 10$ (solid line). The panels (a) and (b) correspond respectively to the real and imaginary part of K_T.

IV. APPLICATION TO ULTRASONIC WAVES

It is known that, when the frequency of a sound wave becomes comparable with the collision frequency of particles, the Chapman-Enskog and 13-moment Grad's methods give poor accounts of the dispersion relation. In the literature several theoretical models [12,13,22–26] have been reported to calculate the dispersion relation of sound waves. In Ref. [22], Sirovich and Thurber have computed the dispersion relation of sound waves, using the hard-sphere model. The comparison with the experimental measurements has shown that their hydrodynamic model is not able to capture the propagation of the sound mode in the high-Knudsen number regime. In Ref. [13], the fluid equation model developed by the authors was found to give good agreement with the experimental results only for the phase velocity of the sound waves. Further generalized hydrodynamic models [12,24,25]

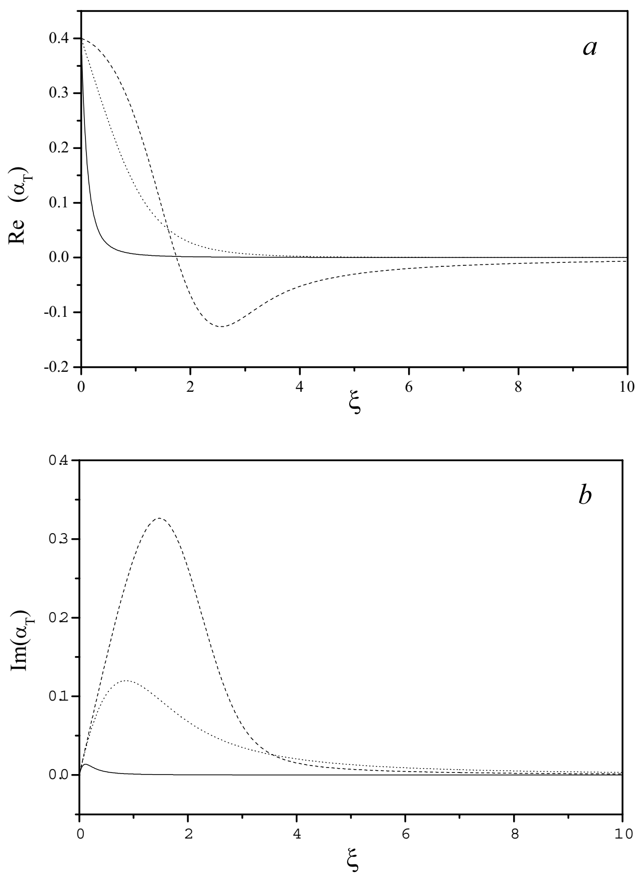

FIG. 2. Normalized temperature anisotropy coefficient α_T as a function of the normalized phase velocity $\xi=\frac{\omega}{\sqrt{2}kv_t}$ for different collisionality parameter $\frac{\nu}{\omega}=0.1$ (dashed curve), $\frac{\nu}{\omega}=1$ (dotted line), and $\frac{\nu}{\omega}=10$ (solid line). The panels (a) and (b) correspond respectively to the real and imaginary part of α_T.

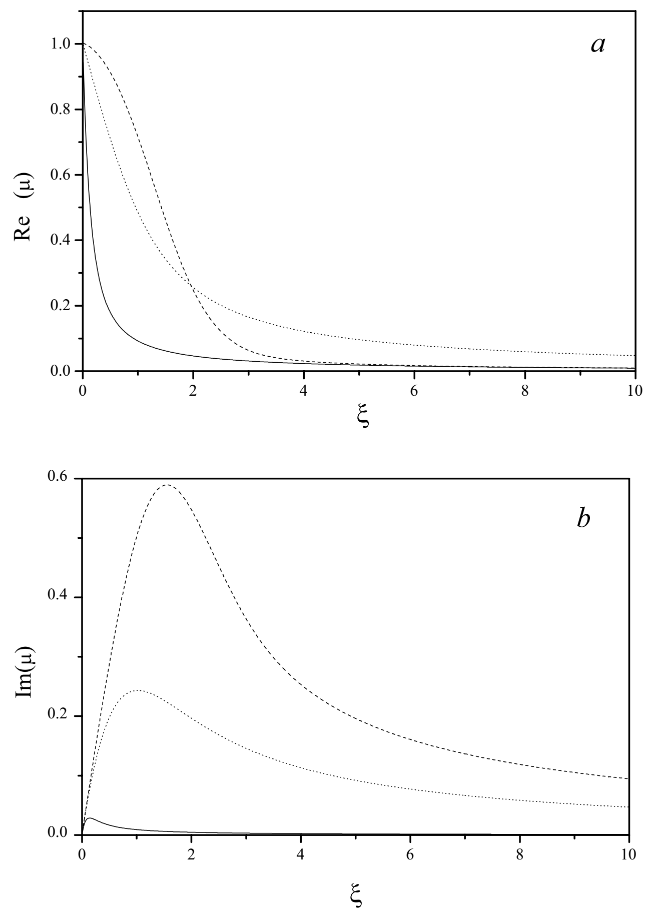

FIG. 3. Normalized viscosity coefficient μ as a function of the normalized phase velocity $\xi=\frac{\omega}{\sqrt{2}kv_t}$ for different collisionality parameter $\frac{\nu}{\omega}=0.1$ (dashed curve), $\frac{\nu}{\omega}=1$ (dotted line), and $\frac{\nu}{\omega}=10$ (solid line). The panels (a) and (b) correspond respectively to the real and imaginary part of μ.

have been also reported in the literature. To test the accuracy of these models, applications to the propagation of sound waves are presented and the results obtained give poor agreements with the experimental data [27–29] in the weakly collisional range. To our knowledge the more accurate results derived in the literature concerning the attenuation and the dispersion of sound waves in weakly collisional gases are due to Sukhorukov and Stubbe [23] and Marques [26].

Sukhorukov and Stubbe have proposed a kinetic model based on the Boltzmann equation with a collision relaxation model derived in Ref. [30]. They treated the problem of sound waves generated by an oscillating boundary by fitting the interval (x_2-x_1) used in the experiments [28], and they found an almost good agreement with the experimental data.

Marques proposed a new extended kinetic description for monatomic gases that is compatible with Grad's 35-moment approximation for monatomic gases of Maxwellian particles, i.e., the interaction forces vary as $1/|\vec{r}|^5$. The kinetic model is based on the Boltzmann equation with a relaxation collision operator $C(f)=-\sigma(f-f_r)$, where σ is an effective collision frequency, $f_r=f_M(A+A_iv_i+A_{ij}v_iv_j+A_{ijk}v_iv_jv_k+A_{ijkl}v_iv_jv_kv_l)$ is a reference distribution function, and A, A_i, A_{ij}, A_{ijk}, and A_{ijkl}, are space and time-dependent coefficients. The theoretical predictions are in very good agreement with the experimental data [27–29]. The most important question in the use of the moment method is, how many moments are needed to describe the physics of a problem accurately, in the sense that the moment equations give a result close to a solution of the Boltzmann equation. For this purpose Marques has also reported results given by the 13-moment and 20-moment approximations. Although these approximations give good agreement with experimental data in the range, $r>1$, where r is the rarefaction parameter, they fail to reproduce the experimental data in the weakly collisional range, $r\ll 1$. This, has shown that for $r\ll 1$, it is necessary to take into account more kinetic effects and they are correctly accounted for by the 35-moment method. We note also that more recently, the kinetic model of Marques was generalized to two-fluid hydrodynamic by Fernandes and Marques in Ref. [31], to study the sound wave properties in binary mixture.

In this section we use the generalized fluid equations derived in the present theory to compute the dispersion relation of free sound waves in the whole collisionality range. To calculate the dispersion relation of a sound wave, we start from Eqs. (2)–(4) linearized with respect to the global equi-

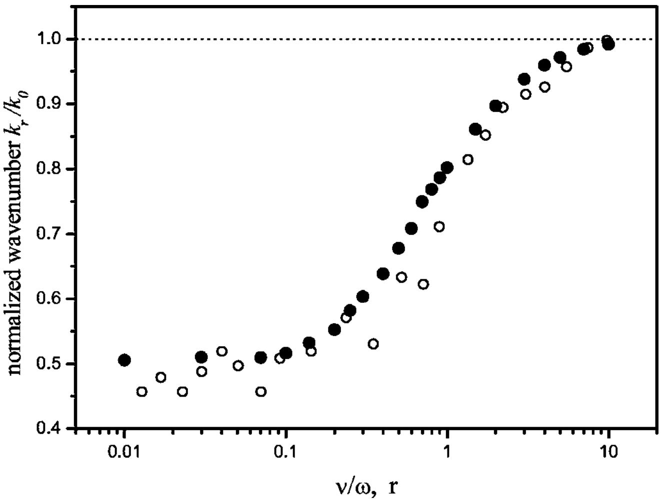

FIG. 4. Wave number (solid circles) k_r/k_0 normalized to the collisional value, as a function of ν/ω. The experimental data of Meyer and Sessler (open circles) are represented as a function of r (the inverse of the Knudsen number). The dotted curve corresponds to the fluid limit.

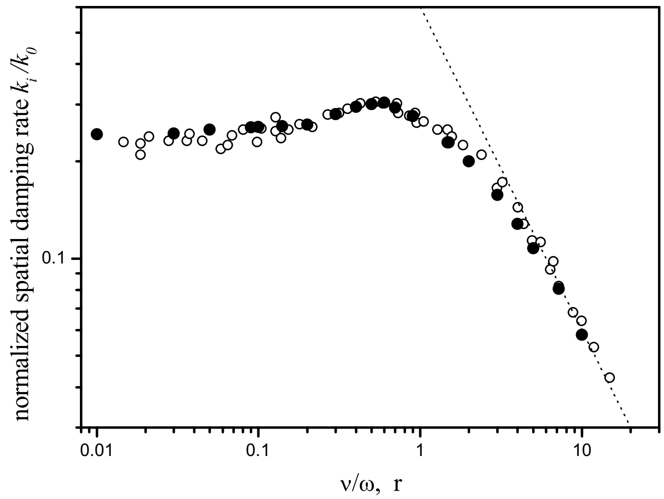

FIG. 5. Normalized spatial damping rate (solid circles) k_i/k_0, as a function of ν/ω. The experimental data of Meyer and Sessler (open circles) are represented as a function of r (the inverse of the Knudsen number). The dotted curve corresponds to the fluid limit.

librium. In the Fourier space $(x\leftrightarrow k, t\leftrightarrow\omega)$, they read

$$-i\omega\tilde{n}(\omega,k)+ikn_0\tilde{V}(\omega,k)=0, \tag{32}$$

$$-i\omega\tilde{V}(\omega,k)=-ik[n_0\tilde{T}(\omega,k)+T_0\tilde{n}(\omega,k)]/n_0m - ik\tilde{\Pi}_{xx}(\omega,k)/n_0m, \tag{33}$$

$$-i\omega\frac{3}{2}[n_0\tilde{T}(\omega,k)+T_0\tilde{n}(\omega,k)]=-ik\tilde{q}_x-ik\frac{5}{2}n_0T_0\tilde{V}(\omega,k). \tag{34}$$

The set of equations (32)–(34) and (B4)–(B7) provides a self-consistent hybrid fluid/kinetic description of the hydrodynamic perturbations in monatomic neutral gases.

The determinant of the algebraic Eqs. (32)–(34) gives the desired dispersion relation for sound waves in neutral gases

$$\omega^2=\Gamma(\omega/kv_t,\nu/\omega)k^2v_t^2, \tag{35}$$

where

$$\Gamma=1+\frac{\frac{2}{3}(1-\alpha_V)(1-\alpha_T)}{1+i\frac{2}{3}\frac{k^2\mathrm{v}_t^2}{\omega^2}\frac{\omega}{\nu}K_T}-i\frac{\omega}{\nu}\mu \tag{36}$$

is the polytropic coefficient. To solve numerically the dispersion relation (35), we proceed as follows: the integrals $Y_{n,m}^{i,j}$ (see Appendix B) involved in the transport coefficients K_T, α_V, α_T, and μ are computed with standard numerical methods and the polytropic coefficient Γ, is deduced. For each value of $\frac{\nu}{\omega}$, the function $\Gamma\left(\frac{\omega}{k\mathrm{v}_t}\right)$ is fitted very accurately by using the ratio of polynomials, $\Sigma_{n=0}^{n=12}a_n\left(\frac{\omega}{k\mathrm{v}_t}\right)^n/\Sigma_{n=0}^{n=12}b_n\left(\frac{\omega}{k\mathrm{v}_t}\right)^n$. Then, Eq. (35) is solved for a real frequency ω and a complex wave number, $k=k_r+ik_i$, since the spatial evolution problem is of great relevance for experiments [27–29]. The numerical results obtained for k_r and k_i are displayed respectively in Figs. 4 and 5.

As a benchmark test for our theoretical model we first consider the limit of strong collisions, i.e., $\frac{kv_t}{\nu}\sim\frac{\omega}{\nu}\sim\varepsilon\ll 1$. In this limit we easily deduce at the lowest order on $\frac{\omega}{\nu}$, $\Gamma=\frac{5}{3}-i\frac{\omega}{\nu}\left(\frac{4}{3}+\frac{10}{9}\frac{k^2v_t^2}{\omega^2}\right)$. Solving iteratively Eq. (35) we obtain the dispersion relation, $(k_r/k_0)^2=1$ and $k_i/k_0=\frac{3}{5}\frac{\omega}{\nu}$ (dotted curves in Figs. 4 and 5) where $k_0=\sqrt{\frac{3}{5}}\frac{\omega}{v_t}$ is the collisional wave number (normalization used in Ref. [28]). The results agree well with the experimental values for a normalized collision frequency comparable to the experimental parameter r (the inverse of the Knudsen number) of Meyer and Sessler, i.e., $\nu/\omega\approx r$. As shown in Figs. 4 and 5, the departure from the fluid theory corresponding to the onset of the kinetic effects, becomes significant when roughly $\frac{\nu}{\omega}<5$. In this case the isotropic function f_0 is not close to the perturbed Maxwellian since the thermal-diffusion time becomes comparable to the hydrodynamic time. The transport coefficients turn out to be nonlocal, i.e., they depend in the Fourier space on the variables ω and k. Since the particles responsible of the transport (the heat-carrying particles for the heat flux for instance) cannot thermalize instantaneously with the thermal population, it results in a reduction of the dissipative transport coefficients and therefore a reduction of the damping rate with respect to the collisional value.

We should expect that as the frequency collision is reduced, the wave number tends to zero (no propagation) and the energy of the sound wave could not be dissipated in the gas $(k_i\rightarrow 0)$. In contrast, the experimental data show that the spatial damping rate diminishes slightly from $\frac{\nu}{\omega}\approx 0.6$ and tends to a constant value $\frac{k_i}{k_0}\approx 0.25$ for $\frac{\nu}{\omega}<0.1$. Correlatively in the weakly collisional range, the wave number remains constant and equal to $\frac{k_r}{k_0}\approx 0.5$. This behavior obviously could not be explained by the use of the collision mechanisms which play an insignificant role in low pressure gases. In these physical situations the perturbations are dissipated mainly by free particle flow since these motions are uncorrelated.

V. DISCUSSION AND SUMMARY

We have considered nonlocal transport processes in dilute neutral monatomic gases when the collision frequency ν and the mean-free path λ are arbitrary with respect to the characteristic frequency ω and the characteristic scale length L, respectively. Because the standard Chapman-Enskog [14] methods are essentially an asymptotic expansion of the total distribution function in small parameters, $(\omega/\nu,\lambda/L)\ll 1$, it cannot be used to describe the system for the most general collisionality regime. The method for solving the kinetic equation developed in this work consists to transform the infinite system of equations into a single equation with the use of the infinite continued fractions. In addition the mathematical techniques of the projection operators are used to ensure the conservative properties of the collision operator. The resulting transport coefficients are complicated functions of the temperature and the flow velocity in the Fourier space (ω,k) and converted back to the space-time space they become integrodifferential operators. Two new transport coefficients (α_V,α_T) that involve only nonlocal effects are calculated. In contrast to the Grad's approach, the present method is not based on a truncation scheme. The whole anisotropic part of the distribution function is kept with the use of the continued fractions.

One of the most important results of this work is the possibility to account for the kinetic effects within the hydrodynamic equations. Such effects are important for instance in dilute low pressure gases, where the collision mechanisms are dominated by the nonlocal effects. As an application of the present work, the dispersion and the attenuation properties of sound waves in monatomic neutral gases is studied in the most general ordering, i.e., for arbitrary values of the relevant parameters $\frac{\nu}{kv_t}$ and $\frac{\omega}{kv_t}$. The results obtained are in very good agreement with experimental data in the whole collisionality range.

In the present approach we use the kinetic equation linearized for small deviations from the equilibrium. Thus, the derived transport coefficients are strictly speaking valid only for the linear case. This is one of the most significant limitations of our theory. Nevertheless, our nonlocal results represent a significant improvement over the standard (local) transport coefficients which are independent on the wave number and the frequency contrary to the nonlocal case. In addition the linear transport theory has been frequently applied successfully outside the limits of its validity. In particular the transport coefficients are habitually computed in the linear approximation [32] and are used as closure relations in nonlinear fluid equations. For instance, the perturbative approach was used by Vidal *et al.* [33] to study the shock waves. The results obtained with the fluid equations closed with nonlocal transport coefficients in the perturbation theory are in good agreement with the Fokker-Planck simulation. On the other hand, experimental observations [34] have been also successfully explained with the nonlocal heat transport theory for small amplitude perturbations.

ACKNOWLEDGMENTS

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APPENDIX A: CALCULATION OF THE PROJECTION OPERATORS FOR THE RELAXATION COLLISION OPERATOR

The purpose of this Appendix is to incorporate in the relaxation collision operator (11) the conservative properties of the collision operator [Eqs. (7) and (9)]. In this paper we use an approach based on the projection operator techniques. This procedure is standard in nonequilibrium statistical physics (see for instance Refs. [16,24]).

We expand the distribution function on an "hydrodynamic part" $P\tilde{f}$ and a "kinetic part" $Q\tilde{f}$ with the use of the projection operator P and its complement orthogonal Q, defined by, $P+Q=1$ and $PQ=0$. As a consequence of this splitting, we can separate the kinetic equation (1) that we express formally as

$$L(f)=C(f) \tag{A1}$$

into a hydrodynamic part (multiplying it by P)

$$P[C(f)]=0 \tag{A2}$$

and a kinetic part (multiplying it by Q)

$$Q[L(f)]=C(f). \tag{A3}$$

The mathematical derivation of the projection operators for the relaxation collision operator (11) is given explicitly in Ref. [16]. We just summarize the different stages of this calculation. First the projection operator P is computed from the isotropic equation (15) using the relation (A2). For this, such as the perturbed Maxwellian, the hydrodynamic part of distribution function is linearly expanded as

$$P\tilde{f}_0(v,k,\omega)=a\exp(-y)+by\exp(-y). \tag{A4}$$

The coefficients a and b are in turn linearly expanded on the hydrodynamic variables $M_0^{1/2}$ and $M_0^{3/2}$ [or equivalently on $\tilde{n}(k,\omega)$ and $\tilde{T}(k,\omega)$]. By means of the usual property of the projection operator, $P^2=P$, we can deduce explicitly the expression of the projector P

$$P(\tilde{f}_0)=\frac{1}{\Gamma(1/2)}\left(\frac{5}{2}M_0^{1/2}-M_0^{3/2}\right)\exp(-y) + \frac{1}{\Gamma(5/2)}\left(-\frac{3}{2}M_0^{1/2}+M_0^{3/2}\right)y\exp(-y), \tag{A5}$$

where $\Gamma(x)$ is the Euler function. We can easily check that $P(\tilde{f}_0)=\tilde{f}_M$ and that, $M_0^{1/2}=M_M^{1/2}$, $M_0^{3/2}=M_M^{3/2}$, where

$M_M^n = \int_0^\infty y^n f_M(y)dy$. Thus, the conservative properties of the collision operators (7) and (9) are well verified.

Equation (A2) is therefore equivalent to the conservative relations (7) and (9), since the contributions of the collisions are removed. The kinetic equation (A1) together with Eq. (A2) are equivalent to Eq. (A3). Since the collisional invariance properties (7) and (9) are isotropic equations, it results that Eq. (A3) is obtained by multiplying Eq. (17) by $Q=1-P$.

To complete our analysis, we should note that to take into account the invariance properties of the collision operator in the kinetic equation, an alternative approach based on the initial value problem was given in Ref. [35] and it yields the same results.

APPENDIX B: EXPRESSIONS OF THE TRANSPORT COEFFICIENTS AS FUNCTIONS OF THE GENERALIZED THERMODYNAMIC FORCES

From Eqs. (26)–(29) we can easily deduce the expressions of the x component of the heat flux and the x-x component of the stress tensor. Multiplying Eq. (26) by y and y^2, integrating over y, we derive a closed set of equations with respect to M_1^1 and M_1^2. Hence we deduce the explicit expressions of M_1^1 and M_1^2 with respect to the hydrodynamic variables, $\tilde{n}(k,\omega)$, $\tilde{T}(k,\omega)$, and $\tilde{V}(k,\omega)$. From (26) we obtain the expression of heat flux

$$\begin{aligned}\tilde{q}_x = &- n_0 T_0 v_t \frac{ik}{|k|}\frac{\tilde{n}}{n_0}\left\{\frac{3}{2}\sqrt{\frac{\pi}{2}}\frac{1}{\Delta}\left[\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right) - \frac{3}{2}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right)\right] - \frac{3}{\sqrt{2}}(-i\xi + \tilde{\nu})\right\}\\ &- n_0 T_0 v_t \frac{ik}{|k|}\frac{\tilde{T}}{T_0}\left\{-\frac{9}{4}\sqrt{\frac{\pi}{2}}\frac{1}{\Delta}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right) - \frac{3}{\sqrt{2}}(-i\xi+\tilde{\nu})\right\}\\ &- n_0 T_0 \tilde{V}\frac{1}{\Delta}\left\{i\xi\left[Y^{1,0}_{5/2,1}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right) - Y^{1,0}_{3/2,1}\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right)\right]\right\}\\ &- n_0 T_0 \tilde{V}\frac{1}{\Delta}\left\{\frac{4}{15}\left[Y^{1,1}_{5/2,1}\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right) - Y^{1,1}_{7/2,1}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right)\right]\right\}. \end{aligned}\tag{B1}$$

On the other hand multiplying Eq. (27) by $y^{3/2}$, using the explicit expressions of the moments M_1^1, M_1^2, and Eq. (29), we obtain the component of the stress tensor

$$\begin{aligned}\tilde{\Pi}_{xx} = &-\frac{8}{45}\frac{1}{\Delta}T_0\tilde{n}\left\{Y^{1,1}_{7/2,1}\left(-\frac{3}{2}Y^{0,0}_{1/2,1} + Y^{0,0}_{3/2,1}\right) - Y^{1,1}_{5/2,1}\left(-\frac{3}{2}Y^{0,0}_{3/2,1} + Y^{0,0}_{5/2,1}\right)\right\} - \frac{4}{15}\frac{1}{\Delta}n_0\tilde{T}(Y^{1,1}_{5/2,1}Y^{0,0}_{3/2,1} - Y^{1,1}_{7/2,1}Y^{0,0}_{1/2,1})\\ &- \frac{16}{135}\sqrt{\frac{2}{\pi}}n_0 T_0 \frac{ik}{|k|}\frac{\tilde{V}}{v_t}i\xi\left\{\frac{1}{\Delta}[Y^{1,1}_{5/2,1}(Y^{0,0}_{3/2,1}Y^{1,0}_{5/2,1} - Y^{0,0}_{5/2,1}Y^{1,0}_{3/2,1}) + Y^{1,1}_{7/2,1}(Y^{0,0}_{3/2,1}Y^{1,0}_{3/2,1} - Y^{0,0}_{1/2,1}Y^{1,0}_{5/2,1})] - Y^{2,1}_{7/2,1}\right\}\\ &- \frac{64}{2025}\sqrt{\frac{2}{\pi}}n_0 T_0 \frac{ik}{|k|}\frac{\tilde{V}}{v_t}\left\{\frac{1}{\Delta}[Y^{1,1}_{5/2,1}(Y^{0,0}_{5/2,1}Y^{1,1}_{5/2,1} - Y^{0,0}_{3/2,1}Y^{1,1}_{7/2,1}) - Y^{1,1}_{7/2,1}(Y^{0,0}_{3/2,1}Y^{1,1}_{5/2,1} - Y^{0,0}_{1/2,1}Y^{1,1}_{7/2,1})] + Y^{2,2}_{9/2,1}\right\}\\ &- \frac{16}{45}\sqrt{\frac{2}{\pi}}n_0 T_0 \frac{ik}{|k|}\frac{\tilde{V}}{\mathrm{v}_t}\tilde{\nu}Y^{1,1}_{5/2,0}. \end{aligned}\tag{B2}$$

In Eqs. (B1) and (B2) we have used the notations $\xi = \frac{\omega}{\sqrt{2}|k|v_t}$, $\tilde{\nu} = \frac{\nu}{\sqrt{2}|k|v_t}$, $\Delta = \sqrt{2}|k|v_t(Y^{0,0}_{3/2,1}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}Y^{0,0}_{1/2,1})$, $Y^{i,j}_{n,m} = \int_0^\infty y^n \tilde{F}_0^m \tilde{F}_1^i \tilde{F}_2^j dy$ and the dimensionless continued fractions

$$\tilde{F}_n = \left[-i\xi + \tilde{\nu} + \frac{(n+1)^2}{[4(n+1)^2 - 1]}y\tilde{F}_{n+1}\right]^{-1}.\tag{B3}$$

With the conservative momentum property [Eq. (8)] which corresponds for the relaxation operator, to the condition $M_1^1 = 0$, we calculate the expression of the density $\tilde{n}(k,\omega)$ as functions of the temperature and the flow velocity. Substituting this expression in Eqs. (B1) and (B2) we readily obtain the generalized dimensionless transport coefficients defined in Eqs. (30) and (31)

$$K_T = \left\{ -\frac{9}{4}\sqrt{\frac{\pi}{2}}\frac{1}{\Delta}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right) - \frac{3}{\sqrt{2}}(-i\xi + \tilde{\nu}) \right\} - C_2/C_1 \left\{ \frac{3}{2}\sqrt{\frac{\pi}{2}}\frac{1}{\Delta}\left[\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right) - \frac{3}{2}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right)\right] - \frac{3}{\sqrt{2}}(-i\xi + \tilde{\nu}) \right\}, \tag{B4}$$

$$\alpha_V = \frac{1}{\Delta}\left\{ i\xi\left[Y^{1,0}_{5/2,1}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right) - Y^{1,0}_{3/2,1}\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right)\right]\right\} + \frac{1}{\Delta}\left\{ i\xi\left[Y^{1,0}_{5/2,1}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right) - Y^{1,0}_{3/2,1}\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right)\right]\right\} - C_3/C_1\left\{ \frac{3}{2}\sqrt{\frac{\pi}{2}}\frac{1}{\Delta}\left[\left(\frac{5}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right) - \frac{3}{2}\left(\frac{5}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right)\right] - \frac{3}{\sqrt{2}}(-i\xi + \tilde{\nu})\right\}, \tag{B5}$$

$$\alpha_T = \frac{4}{15}\frac{1}{\Delta}(Y^{1,1}_{5/2,1}Y^{0,0}_{3/2,1} - Y^{1,1}_{7/2,1}Y^{0,0}_{1/2,1}) - \frac{8}{45}\frac{1}{\Delta}\left\{ Y^{1,1}_{7/2,1}\left(-\frac{3}{2}Y^{0,0}_{1/2,1} + Y^{0,0}_{3/2,1}\right) - Y^{1,1}_{5/2,1}\left(-\frac{3}{2}Y^{0,0}_{3/2,1} + Y^{0,0}_{5/2,1}\right)\right\} C_2/C_1, \tag{B6}$$

$$\mu = \frac{16}{135}\sqrt{\frac{2}{\pi}}i\xi\left\{ \frac{1}{\Delta}[Y^{1,1}_{5/2,1}(Y^{0,0}_{3/2,1}Y^{1,0}_{5/2,1} - Y^{0,0}_{5/2,1}Y^{1,0}_{3/2,1}) + Y^{1,1}_{7/2,1}(Y^{0,0}_{3/2,1}Y^{1,0}_{3/2,1} - Y^{0,0}_{1/2,1}Y^{1,0}_{5/2,1})] - Y^{2,1}_{7/2,1}\right\} + \frac{64}{2025}\sqrt{\frac{2}{\pi}}\left\{ \frac{1}{\Delta}[Y^{1,1}_{5/2,1}(Y^{0,0}_{5/2,1}Y^{1,1}_{5/2,1} - Y^{0,0}_{3/2,1}Y^{1,1}_{7/2,1}) - Y^{1,1}_{7/2,1}(Y^{0,0}_{3/2,1}Y^{1,1}_{5/2,1} - Y^{0,0}_{1/2,1}Y^{1,1}_{7/2,1})] + Y^{2,2}_{9/2,1}\right\} + \frac{16}{45}\sqrt{\frac{2}{\pi}}\tilde{\nu}Y^{1,1}_{5/2,0} + \frac{8}{45}\frac{1}{\Delta}\left\{ Y^{1,1}_{7/2,1}\left(-\frac{3}{2}Y^{0,0}_{1/2,1} + Y^{0,0}_{3/2,1}\right) - Y^{1,1}_{5/2,1}\left(-\frac{3}{2}Y^{0,0}_{3/2,1} + Y^{0,0}_{5/2,1}\right)\right\} C_3/C_1, \tag{B7}$$

where

$$C_1 = -\sqrt{\frac{\pi}{2}}\left(3Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1} - \frac{9}{4}Y^{0,0}_{1/2,1}\right)\Big/\Delta + \sqrt{2}(\tilde{\nu} - i\xi),$$

$$C_2 = \frac{3}{2}\sqrt{\frac{\pi}{2}}\left(-Y^{0,0}_{3/2,1} + \frac{3}{2}Y^{0,0}_{1/2,1}\right)\Big/\Delta,$$

$$C_3 = -\frac{8}{45}\left[Y^{1,1}_{5/2,1}\left(\frac{3}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right) - Y^{1,1}_{7/2,1}\left(\frac{3}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right)\right]\Big/\Delta + \frac{2}{3}i\xi\left[Y^{1,0}_{3/2,1}\left(\frac{3}{2}Y^{0,0}_{3/2,1} - Y^{0,0}_{5/2,1}\right) - Y^{1,0}_{5/2,1}\left(\frac{3}{2}Y^{0,0}_{1/2,1} - Y^{0,0}_{3/2,1}\right)\right]\Big/\Delta.$$

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